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The relationship between simple strain and true strain

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The relationship between simple strain and true strain

Simple (or unit) strain is the change in length over the original length, so that for a pressuremeter measuring radius it can be expressed as

$$\xi_s = \frac{r_i - r_o}{r_o} \qquad \text{...[1]}$$

where ξ_s is simple strain
r_i is the current radius of the cavity
r_o is the original radius of the cavity

From equation [1] it follows that

$$\frac{r_i}{r_o} = 1 + \xi_s \qquad \text{...[2]}$$

True (natural, or logarithmic) strain is defined as the sum of each incremental increase in radius divided by the current radius, so

$$\begin{aligned} \xi_t &= \int_{r_o}^{r_i} (1/r)\,dr \\ &= \left[\ln(r)\right]_{r_o}^{r_i} \\ &= \ln(r_i) - \ln(r_o) \\ &= \ln(r_i/r_o) \end{aligned} \qquad \text{...[3]}$$

where ξ_t is true strain

Substituting equation [2] into [3] gives

$$\xi_t = \ln(1 + \xi_s) \qquad \text{...[4]}$$

It is well known that for instruments which measure the radius of the cavity the following expression can be used to derive estimates for shear modulus from the test curve whenever the response from the ground is elastic:

$$G = \frac{1}{2}\left(\frac{r_i}{r_o}\right)\left(\frac{dP}{d\xi_c}\right) \qquad \text{...[5]}$$

where G is the shear modulus
is the change in pressure
ξ_c is cavity strain and is simple strain

This is sometimes expressed in a simplified form as

$$G = \frac{1}{2}\left(\frac{dP}{d\xi_c}\right) \qquad \text{...[6]}$$

but this approximation can only be justified for very small strains.

The multiplier $\frac{r_i}{r_o}$ has the effect of converting an expression in simple strain to one in terms of true strain, as the following argument shows:

Differentiate equation [4] with respect to ξ_s

$$\frac{d\xi_t}{d\xi_s}=\frac{1}{1+\xi_s}$$

$$\therefore\ d\xi_s=d\xi_t(1+\xi_s) \qquad \text{...[7]}$$

Substitute equations [2] and [7] into [5] to give

$$G=\frac{1}{2}(1+\xi_s)\left(\frac{dP}{d\xi_t(1+\xi_s)}\right)$$

which simplifies to

$$G=\frac{1}{2}\left(\frac{dP}{d\xi_t}\right) \qquad \text{...[8]}$$

Hence the simplified version of the shear modulus expression shown in equation [6] is good for all strains as long as true strain is being used. Plotting true strain rather than simple strain makes it easier to compare modulus parameters taken from rebound cycles at different cavity strains, and makes it easier to compare rebound cycles between instruments which strain the soil to different magnitudes.